Choosing the Optimal Mortar for an Infantry Battalion's Mortar Battery with Analytic Hierarchy Process using Multivariate Statistics

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SCOPE OF THE PAPER

The aim of our research was to choose the optimal mortar for the infantry battalion's mortar battery. The comparison was carried out using Analytic Hierarchy Process methodology (AHP) [1]. In our study we describe the application of Multi-Criteria Decision Analysis methods to select optimal military equipment, and we also show other mathematical models which could be useful to solve decision-making problems. During the process of solving this problem, the most important task was the calculation of weights that show the importance of various attributes. Since in this situation there are many decision makers (for example battalion commanders, battery commanders, infantry and artillery officers), we prepared an opinion survey. We asked 32 officers about the importance of attributes and we calculated 32 vectors based on 32 questionnaires. The result was 32 vectors, each of which described one officer's opinion. One of our goals was to calculate the differences among these opinions. In order to do this, we processed these vectors with Multi-Dimensional Scaling, Cluster Analysis and Box Plot Analysis. The research was carried out through the following steps:

1. Mortar selection criteria.
2. Calculating weights.
3. Comparison using AHP.
4. Sensitivity analysis.

MORTAR SELECTION CRITERIA

The medium-term plan of the Hungarian Defence Forces is to acquire mortar batteries for infantry battalions. In the past we used 120 mm mortars exclusively. Regarding the experience of the last decades, the tasks of the armed forces have changed, therefore we wanted to find out what calibre and what type of mortar is most useful today. In view of our own and foreign experience we selected 8 mortars for comparison: 4 of 81 mm and 4 of 120 mm calibre. In order to avoid the impression of commercial advertising, the manufacturers of the mortars cannot be named.

Criteria:
Rate of fire: The frequency with which the mortar can fire its projectiles. [Round/Minute]
Range: The maximum distance of effective target engagement. [m]
Fragment effect: Effective range of fragments. [m]
Displacement time: Time needed for the battery to dispalce from its firing position. [Minute]
Battlefield mobility: Is the mortar man-portable? [Yes/No]

Artillery experts selected these five criteria. They had considered all the necessary criteria and reduced them to these five. This number is very important. If it were too high, it would make modelling the problem too difficult, and would also increase the possibility of errors in the model. We think that more criteria usually do not yield a more correct result. On the other hand, if the number of criteria is too low, it cannot contain enough information. It follows that there is an optimal number of criteria. Based on our former experience we advised the artillery experts to choose fewer than eight criteria [2].

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CALCULATING WEIGHTS

The calculation is based on a survey. We made two types of questionnaires. In the first type (Figure 1) the expert had to make a preference order of the criteria. The second type (Figure 2) was an AHP form which was developed from our criteria. 32 experts completed the questionnaires.

Box Plot Analysis

The first questionnaires were processed by box plot analysis. Figure 3 shows the results. The vertical axis shows the criterion's position in the order. The box plot analysis is a good method to examine the opinions of experts. The boxes show the Interquartill range. The size of this range shows the dispersal of the experts' opinions. If this box is small, the difference s among the experts' opinions is small, too. If we compare the positions of the boxes we can decide the order among the criteria. For example the box of the second criterion (Range) is between 1 and 2. Its median is 1. This means that at least 50% of the experts think that this criterion is the most important but there are many experts who think otherwise. The box of the first criterion (Rate of fire) is between 2 and 3, its median is 2. It means that at least 50% of the experts think that this criterion is second in importance among the criteria. It follows that the majority of experts believe that the range of a mortar is more important than its rate of fire.

If we look at the other boxes we can see that these boxes are bigger than the first two boxes. We cannot clearly determine the order between these criteria. An average cannot be calculated based on the results of the Box Plot Analysis. The differences among the experts' opinions are too high; therefore an average number can not represent all the experts. One possible solution is to group the experts according to the similarity of their opinion concerning a criterion's importance, and generate weights based on the groups.

The result of the first type of questionnaire was simple order. There are some methods which could be useful to group experts with similar opinions, for instance the rank correlation method, but there is a problem. Our goal is to calculate weights. Weights can be calculated based on the arithmetic mean, but cannot be calculated based on the order mean. Therefore we used another type of questionnaire. That was an AHP questionnaire, whose results were ratio scale numbers. These numbers can be expected to have arithmetic mean and this can be the weight.

The results of the Box Plot Analysis show high dispersion among the experts' opinions. Based on it we have to say: there are many different opinions (artillery and other officers and NCOs) about mortars. This statement is confirmed by the last box in the Figure 3. This box shows the range of opinions about the mortar's mobility, and we know that there is a huge difference between the 120 mm mortars and the 81 mm mortars mobility. This is the largest box, and it follows that many experts consider large calibre mortars desirable, but another large group prefers small calibre ones.

In general, our goal was to select the best military equipment to use, therefore the selection had to be based on the user's requirements. But what should we do if these requirements are not clear? According to preliminary results we can assume that the users themselves are not very certain about their own requirements. Which one is the most preferable?

Multivariate Statistics

The second type of questionnaire was based on AHP methodology. We determined the eigenvectors of the 32 experts' 32 Paired Comparison Matrices and checked their consistency. Because of the significance level 13 were acceptable. These results were ratio scale number as AHP methodology.

We used two types of multivariate statistics model: Cluster Analysis or clustering and Multidimensional scaling. Both methods are based on a kind of distance measurement. The methods use the vectors of the experts. Each expert has a five dimensional vector which shows the weights he is assigned and it contains his opinion about the weapon. The methods measure the distance among the vectors and this distance shows the difference among the experts' opinions.

Figure 4 shows the result of clustering. As shown in Figure 4, a hierarchical method was used. Hierarchical algorithms find successive clusters using previously established clusters. Hierarchical

clustering has a divisive algorithm. In the first step each expert forms a cluster therefore in the beginning we have 13 clusters. In the second step the algorithm searches out and groups the experts with the least difference in opinion. The new clusters are formed around this member. In this case these are 2,5; 1,30; 6,31;12; etc. The last step forms a single cluster that has 13 members.

In order to obtain more accurate results, Multidimensional scaling was carried out. Input data of Multidimensional scaling are the vectors of experts. The method of Multidimensional scaling calculated Squared Euclidian Distance between these vectors, and based on it plotted the experts as points in a two dimensional coordinate system. The result of the MDS is 13 points in a plane (see Figure 5). Each point represents an expert. When we examine the result we have to look at the position of the expert. Based on the mathematical model, if two or more experts' points are located close to each other, then the opinions of these experts' are similar. If they are located far from each other, they have different opinions about the mortar.

Figure 5 shows the result of this analysis. Based on the two methods' results we separated 4 subgroups, as follows:

A: S1, S2, S5, S10, S30
B: S6, S9, S12, S31
C: S24, S25
D: S20, S21

There is only one difference between the two results. The two methods put S10 into different groups. If you examine his position on Figure 5, you find that this expert's position is ambiguous. In this case there are two possible solutions. The first one is to eliminate this expert; the second one is to decide his position using other information. We examined his personal vector and based on it we put that expert into Group A.

According to cognitive psychology, an individual makes decision on the basis of an inner scale of values he is often not aware of. For instance, on the questionnaires he may mark range or fragment effect as more important than mobility, because he prefers the much bigger 120 mm mortar to the 81 mm one. Using Multidimensional scaling, these internal factors were established. In this case the Figure 5 axis shows these internal factors. We identified the internal factor based on the bar graphs of Figure 6. Each describes one group's average opinion about the mortars. We calculated it with arithmetic mean. Location of the bar graphs is similar to the result of Multidimensional scaling in Figure 5. We examined the changing of the criteria's weight on the axes.

We can name the first three criteria (Rate of fire, Range, Fragment effect) as firepower. If you look at Figure 6, you can see that mobility increases from left to right. The vertical axis shows the changing of the fire power's three criteria's importance. The range and the fragment effect increase and the rate of fire decrease from bottom to top. If you examine displacement time, you can see that there are no correlations with the axis.

If we examine Groups C and D we can say:
They prefer mobility to firepower. It follows that they want an 81 mm mortar.

If we examine the Groups A and B we can say that:
They prefer firepower to the mobility. It follows that they want a 120 mm mortar.

If you compare the groups in the top and the bottom you can see that the experts on the bottom prefer range and fragment effect to rate of fire. If an artillery piece has great range and high fragment effect but it has medium rate of fire, it is a gun. On the bottom the expert want an artillery piece that has great rate of fire and medium range and fragment effect.

Based on Figure 6 we can say that:
We grouped the experts into four groups. Each group has a separate opinion about the mortar. There is no group that represents average opinion. The right side wants 81 mm and the left side wants 120 mm. The top wants a gun and the bottom wants a mortar. The experts are balanced between medium calibre and small calibre, and they are balanced between the types of artillery. It is an interesting to note, that the experts of Groups C and D are members of the Light Infantry Brigade, which rarely takes part in peace keeping missions. The Group A and B expert are mainly members of the National Defence University and the HQ of the Hungarian Defence Force. There is one result based on current practical experience, and there is another result based on plans and scientific research.

There is a huge difference between the four groups' opinions, but we have to calculate a single weight vector, because it is necessary for further calculations. The average calculation is not recommended because the results do not properly represent the groups.

If we can not determine the average opinion, than we have to determine the opinion of the majority. To select the majority we used the first type of questionnaire. This questionnaire shows order of criteria. This is not suitable for calculating weights but is suitable for identifying the majority. 71% of the 32 experts prefer firepower to mobility. As Figure 6 shows, Groups C and D consist of only 4 members, and Groups A and B consist of 9 members, i.e. 69%. Based on it we declare that Groups A and B show the majority opinion. No distinction was made between the two groups; therefore further calculations of two vectors were used. It follows that we have two outcomes: one for Group A, and one for Group B.

COMPARISON USING AHP

Our 8 mortars were ranked by using the AHP regarding their criteria and their weights. AHP do not use the utility function. The decision maker defines the ratio witch shows the difference between the utility of alternatives. A criterion can be subjective for instance ergonomic or aesthetics. In this case the decision maker directly decides the mentioned ratio. If we can measure a criterion, for instance rate of fire, using these numbers we can determine ratios, and we can make the Paired Comparison Matrices of AHP.

Measurements of the Rate of fire, Range, Fragment effect are based on the 1st point. Battlefield mobility is a true or false criterion, if the mortar is man-portable, it has a value of 1 point, if it is not, then it has a value of 0.

We calculated every mortar's overall score. We used the following expression:

$$x_j = \sum_{i=1}^{m} w_j a_{ij} \frac{1}{\max_j a_{ij}^{HE}} .$$

x_j overall score of mortar number j;
w_i weight of criterion number i;
m number of criteria;
a_{ij} score of mortar number j by criterion number i;
a_{ij}^{HE} score of mortar with High Explosive projectile number j by criterion number i.

Figure 7 and Figure 8 show the overall utility of the mortars. Figure 7 shows the utilities of Group A, and Figure 8 shows the utilities of Group B. Each 120 mm mortar has two points; the first shows its utility with He projectile and the second one shows the utility with Extended Range projectile. If you compare the two figures, it is clear that 120 mm mortars with Extended Range projectile is significantly better than the others therefore, only the High Explosive projectile scores are examined further.

If you examine the result of Group A you can see that the 5th mortar has the highest score. This is a 120 mm one. On this basis it seems that this is the best mortar.

The result of Group B is not clear. The best mortar is the first one and his score of 0.71, but the second best has score of 0.69. The difference is very small, therefore the order is ambiguous. Against this background, we were looking for a method which can check the possibility of change order.

SENSITIVITY ANALYSIS

Using the Multi-Criteria Decision-making model we have no information about the accuracy of the results. It may cause problems if the results, which are single numbers, are close to each other. In this case we calculated intervals, using the following hyperbolic programming, instead of numbers, and by comparing the intervals we have reached convincing results [3].

$$x_j^- = \min_{\mathbf{t}} \sum_{i=1}^{m} \frac{t_i a_{ij}}{\sum_{k=1}^{m} t_k} , \ t_i \in \left[w_i^-, w_i^+ \right], \ i = 1,\dots,m, \ j = 1,\dots,n,$$

$$x_j^+ = \max_{\mathbf{t}} \sum_{i=1}^{m} \frac{t_i a_{ij}}{\sum_{k=1}^{m} t_k} , \ t_i \in \left[w_i^-, w_i^+ \right], \ i = 1,\dots,m, \ j = 1,\dots,n.$$

w_i^+, w_i^- input data that show the change of weight, in this case ±15%;
x_j^+, x_j^- output data that show the change of overall score.

Figure 9 shows the result. The figure shows the intervals of the mortars. The intervals indicate the change of score for every 15% change of weight. These intervals are shown by thick horizontal lines. In order to make it clearer visually, vertical lines run from the ends of the best mortar's interval to the X axis (shown in Figure 9 by a slash pattern square). Examining this square we can see where the other mortars' intervals overlap.

Figure 9 shows sensitivity analysis of the result of Group A. You can see clearly that the 120 mm mortar with Extended Range projectile is far superior to the other types.
If you examine the interval of M5 you can see that this is the best mortar for High Explosive and Extended Range projectiles, but the total order is not clear because the M1, M2 M3 mortars' intervals overlap.

Figure 10 shows sensitivity analysis of the result of Groups B and C. You can see that most intervals overlap. We can not choose the best type.

Group A result and its sensitivity analysis show clearly that M5 is the best and sensitivity analysis of Group B does not preclude the primacy of M5 therefore in this case also the best solution choosing M5 type mortar.

SUMMARY

The goal of the research was to choose the best mortar for the infantry battalion. In order to reach this goal we used Multi Criteria Decision Analysis. The users' requirements were assessed by a survey. Based on the survey and using AHP the best solution was identified, and the accuracy of comparison was checked by sensitivity analysis.
Based on the survey and the processing of data we summarize the following result:
1. Users have different opinions about the capabilities of mortars.
2. The majority prefer a 120 mm mortar.
3. The best solution selected is M5 type mortar which meets most user requirements.

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[3] Mészáros, Cs., Rapcsák, T.: On sensitivity analysis for a class of decision system, Decision Support Systems, 16 (1996) 231-240 p. ISSN: 0167-9236

Put the proper order of importance in the left column points!

Criteria		Your prioritization	
Rate of fire	1		Most important ↑
Range	2		
Fragment effect	3		
Displacement time	4		
Battlefield mobility	5		Least important

Figure 1
1st questionnaires

| Questionnaires																		
1=Equal importance		3= Moderate importance			5= Strong importance				7= Very strong importance				9= Extreme importance					
1	**Rate of fire**	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	**Range**
2	**Rate of fire**	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	**Fragment effect**
3	**Rate of fire**	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	**Displacement time**
4	**Rate of fire**	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	**Battlefield mobility**
5	**Range**	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	**Fragment effect**
6	**Range**	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	**Displacement time**
7	**Range**	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	**Battlefield mobility**
8	**Fragment effect**	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	**Displacement time**
9	**Fragment effect**	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	**Battlefield mobility**
10	**Displacement time**	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	**Battlefield mobility**
Rate of fire		The frequency with which the mortar can fire its projectiles																
Range		The maximum distance of effective target engagement																
Fragment effect		Effective range of fragments																
Displacement time		Time needed for the battery to displace from its firing position																
Battlefield mobility		Ability of carrying out the mortar by manpower on battlefield																

Figure 2
2nd questionnaire
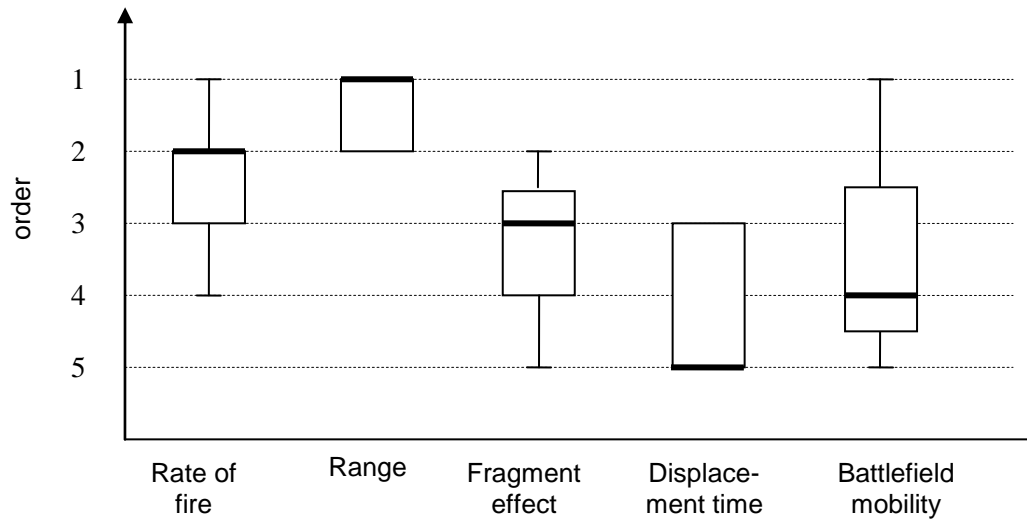
Figure 3
Box Plot Analysis
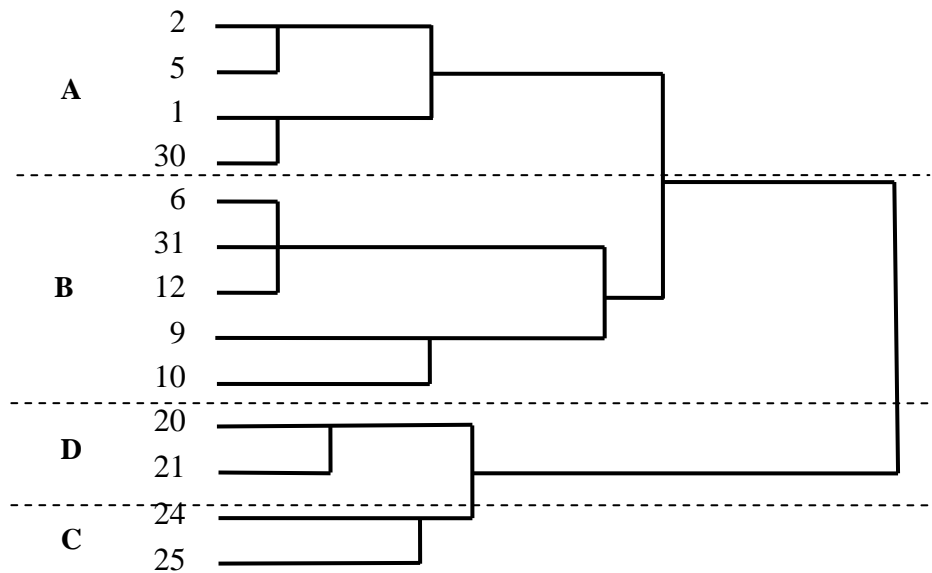
Figure 4
Hierarchical Cluster Analysis
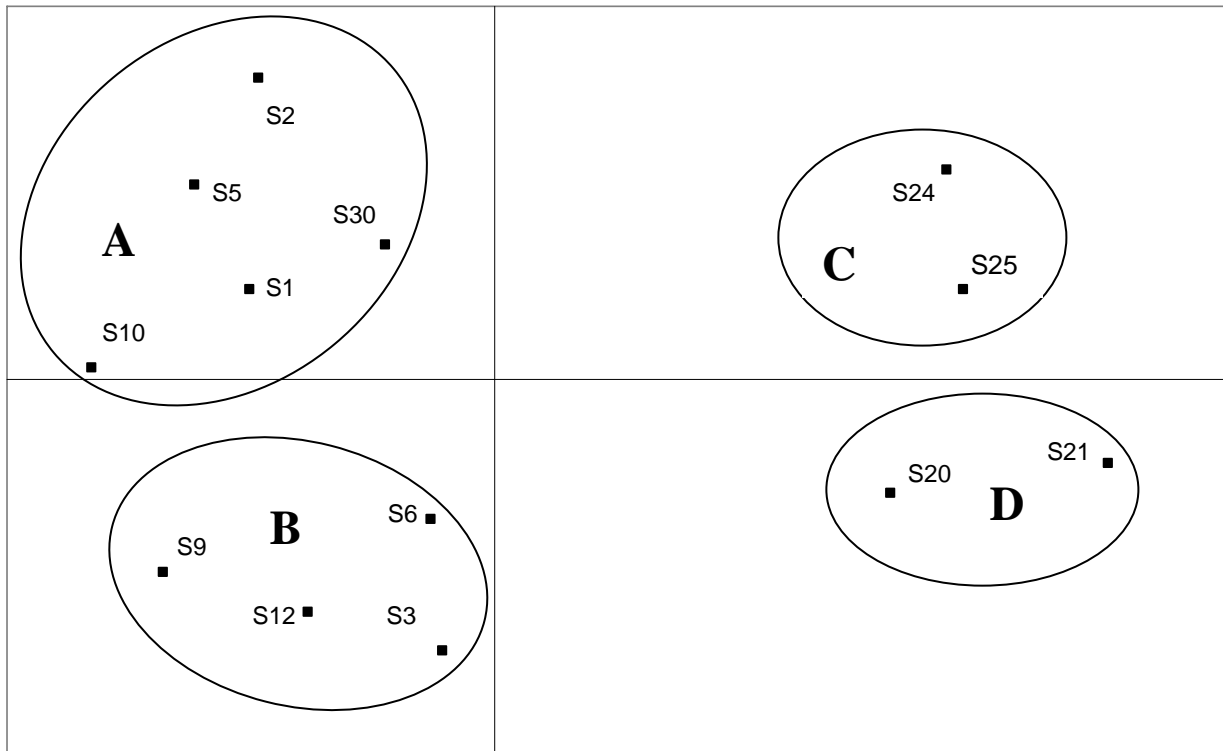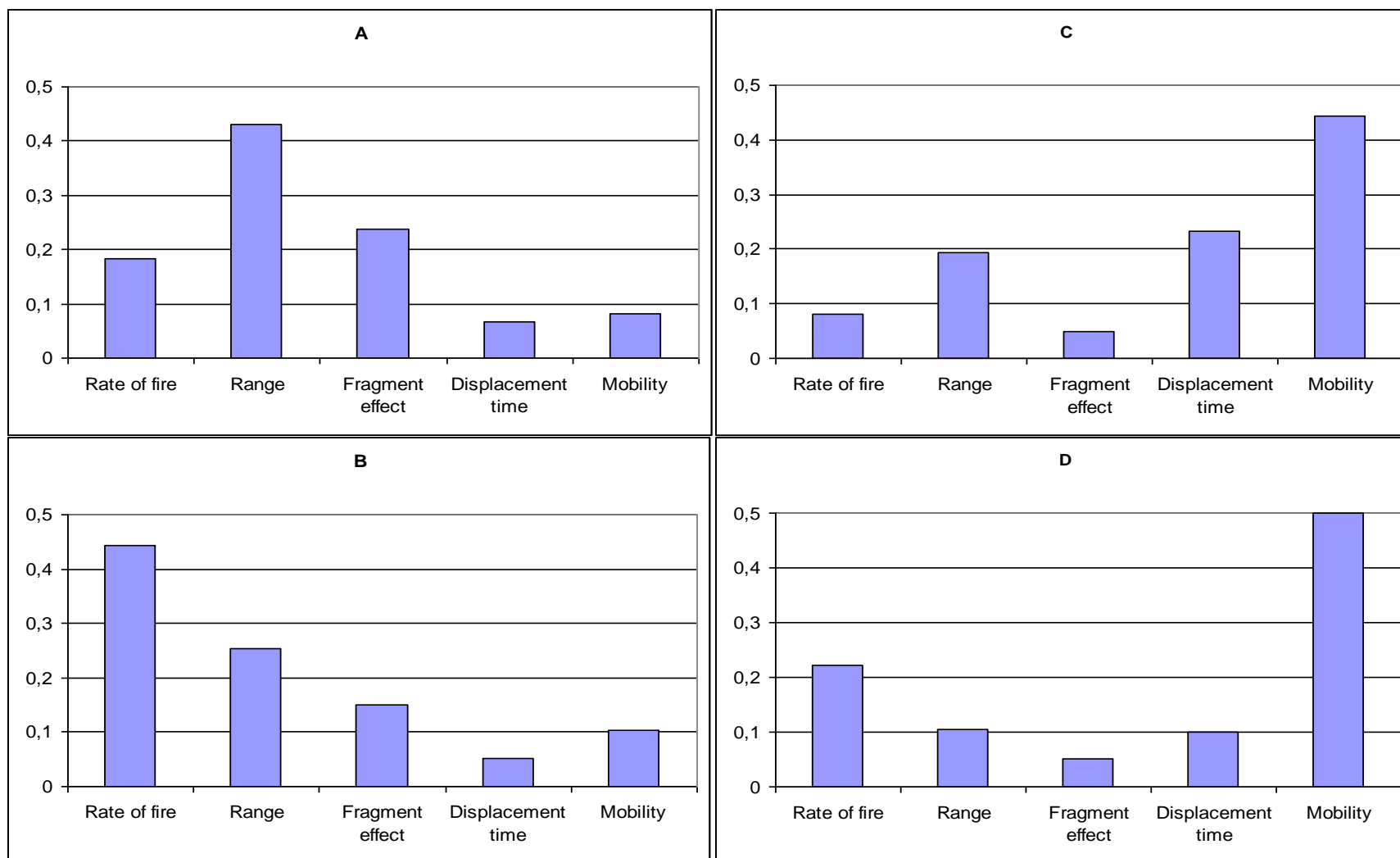
Figure 5
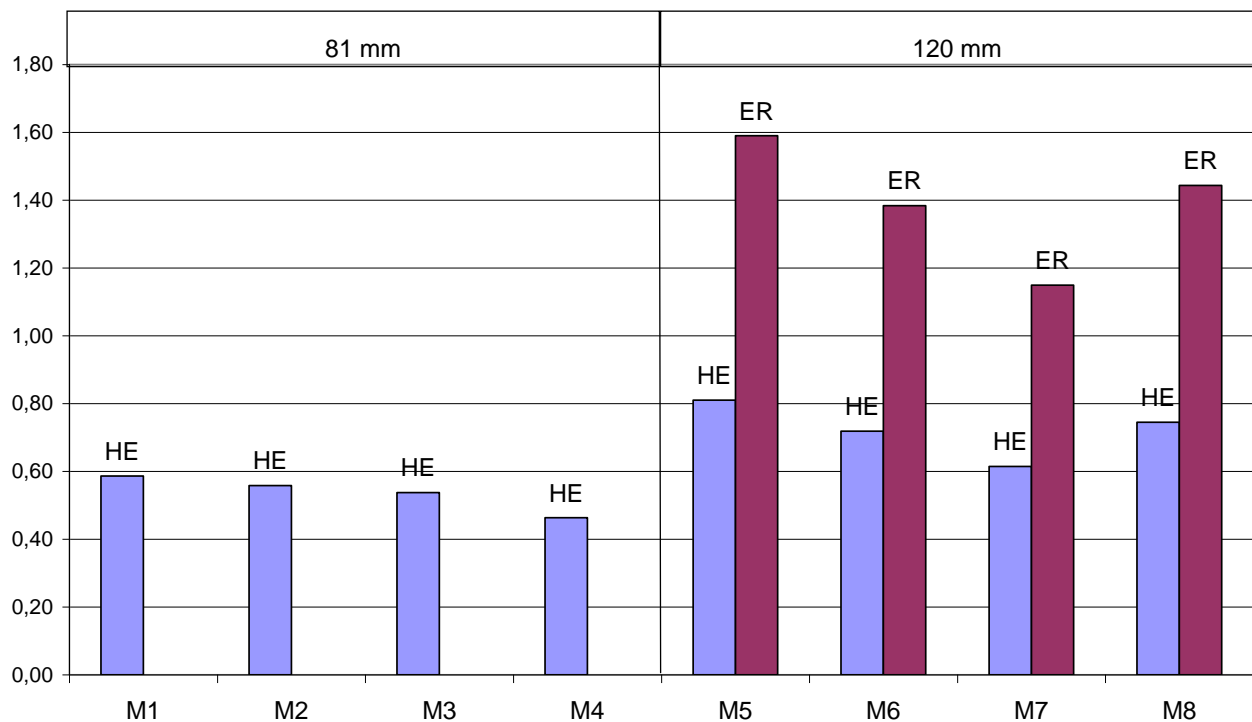Multi Dimensional Scalin

Figure 6
Weights of the Groups

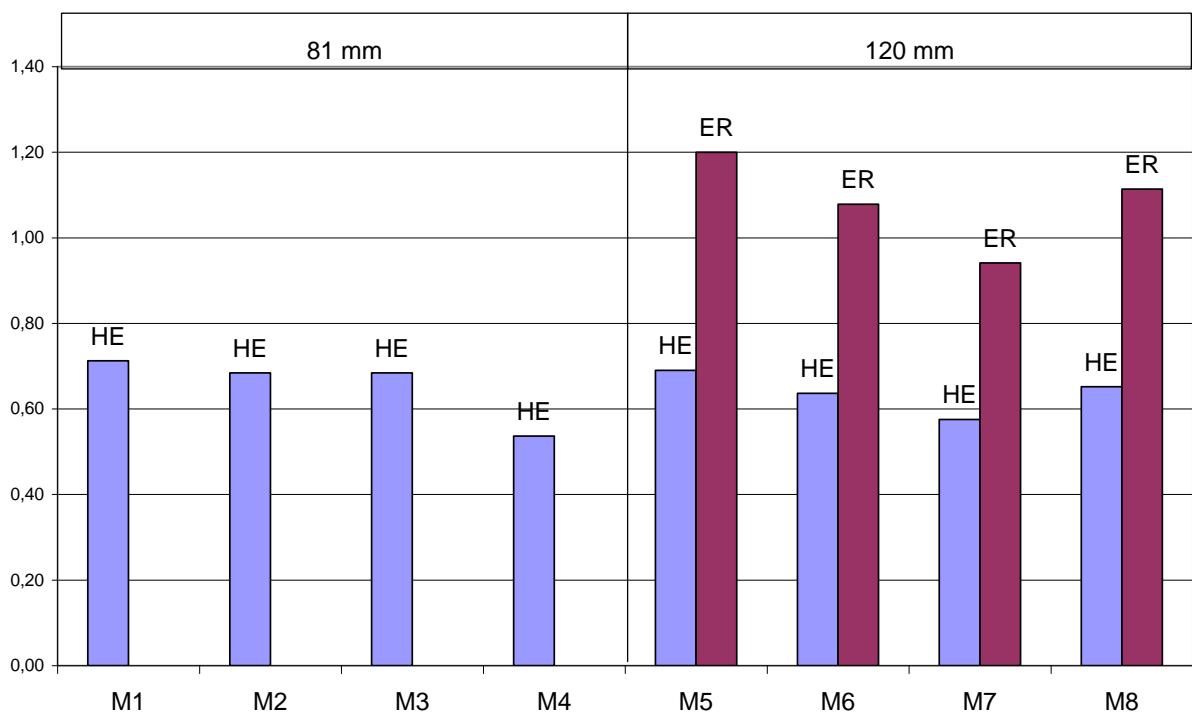Figure 7
Result based on the opinion of Group A

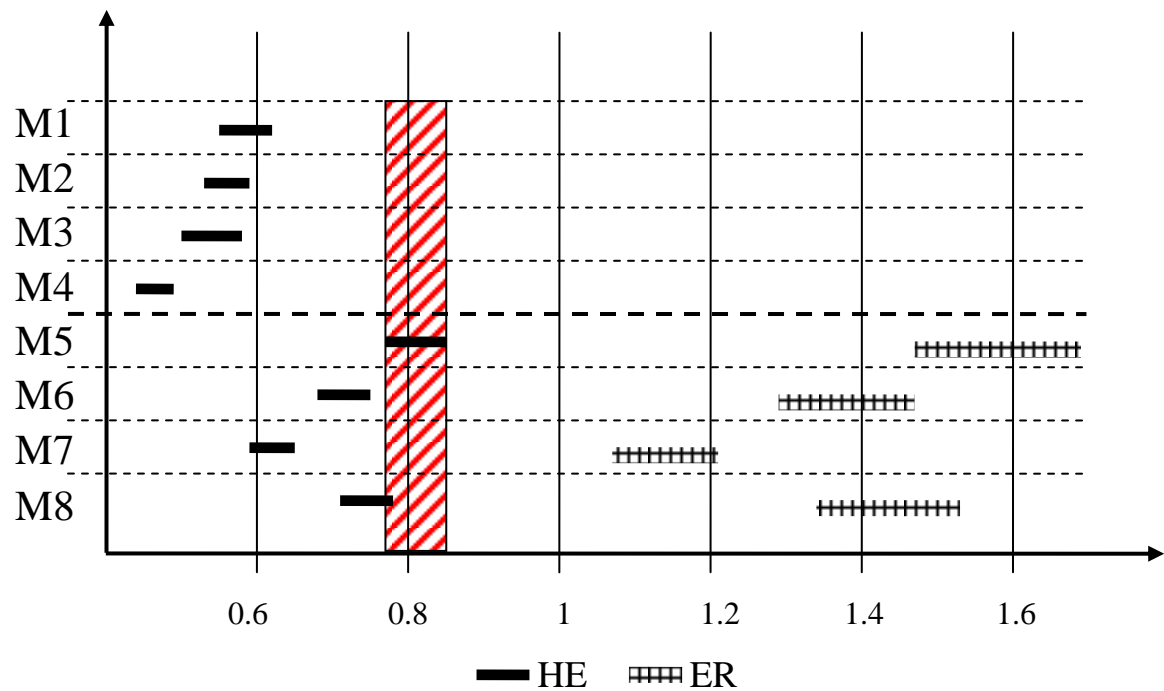

Figure 8
Result based on the opinion of Group B
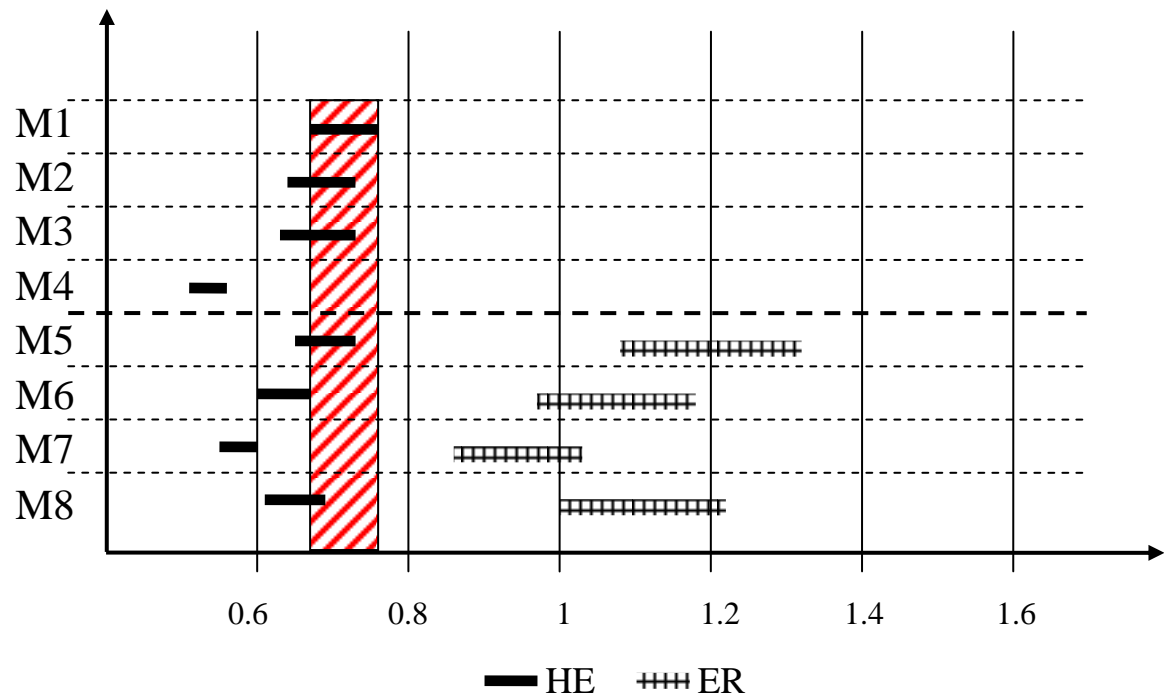
Figure 9
Sensitivity analysis of result of Group A

Figure 10
Sensitivity analysis of result of Group B